TURBULENCE INTENSITY OF AN ASCENDING DISPERSED FLOW

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We obtained functions that approximately describe the change in the relative turbulence intensity of hydrodispersed and gas-dispersed vertical flows and make it possible to explain the different effects of solid particles on flow turbulence. A comparison was made with experimental data on heat exchange, which showed the results to be in qualitative agreement.

The influence of a discrete impurity on flow turbulence has been the subject of numerous experimental and theoretical studies. Theoretical publications [1, 2] have dealt with horizontal laminar turbulent flows. In the present investigation, we determined the turbulence intensity of a monodispersed vertical flow in order to establish the conditions under which the presence of particles can have different effects on flow turbulence.

We assume that the particles are small with respect to the scale of the turbulence and use the following model of a quasihomogeneous flow:

$$\rho_f = \rho(1-\beta) + \rho_T\beta, \tag{1}$$

$$v_{fi} \doteq \frac{\rho(1-\beta)}{\rho_f} v_i + \frac{\rho_T\beta}{\rho_f} v_{Ti}. \tag{2}$$

The only important mass force is the force of gravity and, in view of the smallness of the particles (neglecting their additional inertia), we obtain the following expression for the stabilized component of the ascending flow:

$$v_{Ti} = v_i - v_s\delta_{i1}, \tag{3}$$

where the suspension rate v_s with sufficiently small values for the flow criterion [3] can be regarded as independent of the particle concentration and channel geometry. In this case, the equation of motion for the mixture acquires the following form, taking into account the rule of repeated subscripts [1]:

$$\frac{\partial \rho_f v_{fi}}{\partial \tau} + \frac{\partial}{\partial X_\alpha}(\rho_f v_{fi} v_{f\alpha} + P\delta_{i\alpha} - \sigma_{i\alpha}) = -\rho_f\left[g + \frac{\rho\rho_T}{\rho_f}\frac{\partial}{\partial X_1}\frac{\beta(1-\beta)v_s^2}{\rho_f}\right]\delta_{i1}, \tag{4}$$

where P and $\sigma_{i\alpha}$ are respectively the pressure and viscous-friction stress in the flow. The discontinuity equation

$$\frac{\partial \rho_f}{\partial \tau} + \frac{\partial \rho_f v_{f\alpha}}{\partial X_\alpha} = 0 \tag{5}$$

is readily transformed to

$$\frac{\partial v_{f\alpha}}{\partial X_\alpha} = -(\rho_T - \rho)\frac{\partial}{\partial X_1}\frac{\beta(1-\beta)v_s}{\rho_f}. \tag{5'}$$

The dispersed-medium flow is considered to be steady-state with respect to its average characteristics and to move in one (vertical) direction OX_1. It follows from the stationary condition that there cannot be a transverse flow of mass constant in direction, i.e., the average mass transfer proceeds simply in the direction of the OX_1 axis.

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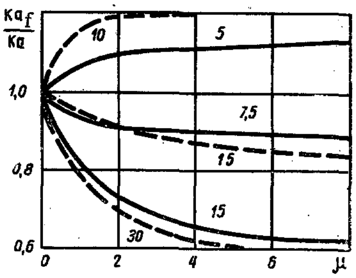

Fig. 1

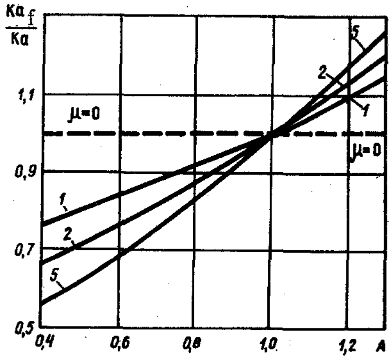

Fig. 2

Fig. 1. Relative turbulence intensity for dispersed flow as a function of solid-particle concentration.

Fig. 2. Relative turbulence intensity of dispersed flow as a function of complex $A = \sqrt{kv/v_t}$.

$$\overline{\rho_f v_{f2}} = \overline{\rho_f v_{f3}} = 0. \tag{6}$$

It then follows from discontinuity equation (5) that $\partial \rho_f v_{f1}/\partial X_1 = 0$, whence it is easy to convert to the usual expression for constancy of mass flow rate:

$$\overline{\rho_f v_{f1}} = \overline{\rho v_1 (1-\beta)} [1 + \mu] = \text{const.} \tag{7}$$

The flow can be treated as homogeneous and symmetric with respect to the vertical axis in the region of quasistabilized motion. It then follows from Eq. (7) and $\partial \bar{v}_{f1}/\partial X_1 = 0$ that

$$\partial \bar{\beta}/\partial X_1 = 0. \tag{8}$$

The symmetry condition for the average flow, taking Eq. (6) into account, yields

$$\bar{v}_{f2} = \bar{v}_{f3} = 0. \tag{9}$$

It follows from Eq. (8) that the last term on the right side of Eq. (4) and the entire right side of Eq. (5') equal zero. Barenblatt [1] neglected the first of these quantities only because of the smallness of β. In this form, these equations are identical to the equation of motion and discontinuity equation for a homogeneous compressed liquid [1]. We can therefore use the turbulence-energy balance equation for such media given by Monin and Yaglom [4]. Assuming both components of the dispersed flow to be incompressible, proceeding from the semiempirical theory of turbulence, and taking into account Eqs. (6), (8), and (9), we can exclude from the balance equation the quantity $\rho_f v'_{fi}$, which is characteristic only of a compressible liquid:

$$\overline{\rho_f v'_{f1}} = \overline{\rho'_f v'_{f1}} = (\rho_т - \rho)\,\overline{\beta' v'_{f1}} = -(\rho_т - \rho) K_\beta \frac{\partial \bar{\beta}}{\partial X_1} = 0. \tag{10}$$

In this case, we can use the appropriate energy balance equation for the flow of an incompressible medium. From the physical standpoint, this means that the energy going to suspension of the particles comes from the energy of average and not pulsational motion. Taking into account the simplifying assumptions made by Monin and Yaglom [4], the energy of pulsating motion is described by the equation

$$\frac{\partial E}{\partial \tau} = -\bar{\rho}_f\, \bar{\varepsilon} - \bar{\rho}_f\, \overline{v'_{f\alpha} v'_{fi}}\, \frac{\partial \bar{v}_{ni}}{\partial X_\alpha}.$$

When we shift to a cylindrical coordinate system and introduce the turbulence-intensity expression

$$b_f = E/\bar{\rho}_f = \frac{1}{2}\, \overline{v'_{f\alpha} v'_{f\alpha}} \tag{12}$$

equation (11) acquires the form

$$\frac{\partial b_f}{\partial \tau} = -\bar{\varepsilon} - \overline{v'_{f1} v'_{fr}}\, \frac{\partial \bar{v}_{f1}}{\partial r}. \tag{13}$$

Using the theory of dimensionality, the specific dissipation of turbulent energy $\bar{\varepsilon}$ can be written in the form

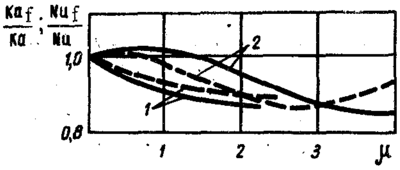

Fig. 3. Relative turbulence intensity and relative heat-transfer rate for dispersed flow as a function of particle concentration.

$$\bar{\varepsilon} = b_f^{3/2}/lc^4,$$

where l is the turbulence scale and c is some dimensionless parameter. Taking into consideration the relationship $v'_{f1}v'_{fr} = -K\partial v_{f1}/\partial r$, where the turbulent-transfer coefficient $K = lb_f^{1/2}$, Eq. (13) has the form

$$\frac{\partial b_f}{\partial \tau} = -\frac{b_f^{3/2}}{c^4 l} + lb_f^{1/2}\left(\frac{\partial \bar{v}_{f1}}{\partial r}\right)^2. \tag{14}$$

The Reynolds equation is used as the formula for the average motion of the entire mixture; near the channel walls, it can be written as

$$K\frac{\partial \bar{v}_{f1}}{\partial r} = v_f^{*^2}. \tag{15}$$

The dynamic velocity of the quasihomogeneous dispersed flow v_f^* can be treated as independent of the coordinates; it is a complex, experimentally determinable function of the properties of the continuous and discrete components and of the average volume concentration. In Barenblatt's study [1], the value of v_f^* in the equation for average motion was assumed to equal the dynamic flow velocity without impurities v^*, which, as will be shown below, cannot always be regarded as justified. It follows from Eqs. (14) and (15) that

$$\frac{\partial b_f}{\partial \tau} = -\frac{b_f^{3/2}}{c^4 l} + \frac{v_f^{*^4}}{lb_f^{1/2}}. \tag{16}$$

Equation (16) then yields the condition for existence of turbulent pulsations "steady-state" with respect to intensity ($\partial b_f/\partial \tau = 0$):

$$b_f = c^2 v_f^{*^2}. \tag{17}$$

Barenblatt [1] found that

$$b_f = c^2 v^{*^2}(1 - \mathrm{Ri}). \tag{17'}$$

The absence of the factor (1 − Ri) in Eq. (17) is due to the fact that it reverts to one for vertical flows, since the dimensionless parameter Ri, like the Richardson number, is proportional to $\partial\bar{\beta}/\partial X_1 = 0$.

The dynamic velocity is determined from the expressions for tangential frictional stresses at the boundaries for homogeneous and dispersed flows:

$$\begin{aligned} \sigma_w &= \lambda\frac{\rho v^2}{8} = \rho v^{*^2}, \\ \sigma_{wf} &= \lambda_f\frac{\rho v^2}{8} = \rho_f v_f^{*^2}. \end{aligned} \tag{18}$$

Then

$$\frac{b_f}{b} = v_f^{*^2}/v^{*^2} = \lambda_f \Big/ \lambda\left[1 + \left(\frac{\rho_T}{\rho} - 1\right)\beta\right]. \tag{19}$$

If we assume $v_f^* = v^*$, as in Barenblatt's article [1], Eq. (19) for vertical flows would then lead us to conclude that the turbulence intensity of the flow is independent of whether or not it contains impurities. This conclusion would probably be valid only for suspensions of more or less uniform density. When the Karman number ($\mathrm{Ka} = b^{1/2}/v$), a relative measure of the turbulent velocity pulsations, is used, the change in this quantity when a discrete impurity is introduced into the flow amounts to

$$\frac{\mathrm{Ka}_f}{\mathrm{Ka}} = \frac{v}{v_\Pi}\left(\frac{b_f}{b}\right)^{1/2} = \frac{\sqrt{[1 + (\rho_T/\rho - 1)\beta]\,\lambda_f/\lambda}}{1 + (\rho_T v_T/\rho v - 1)\beta}. \tag{20}$$

According to Duran [6], vertical flows of hydrosuspensions with a particle diameter of less than 50 μ yield the same pressure loss as pure water if it is expressed as the height of the mixture column, i.e., $\lambda_f/\lambda = \rho_f/\rho$. Since the slip can be neglected for such small particles ($v_t \simeq v$), we readily find from Eq. (20) that $Ka_f/Ka = 1$. When the hydrodynamic flow contains larger particles, the pressure loss tends to its value in pure water [6] and the average density of the mixture no longer plays a material role, i.e., $\lambda_f = \lambda$. Hence

$$\frac{Ka_f}{Ka} = \frac{\sqrt{1 + (\rho_T/\rho - 1)\beta}}{1 + (\rho_T v_T/\rho v - 1)\beta}. \tag{21}$$

This quantity may be only slightly larger or smaller than one, depending on the density of the particle material and the particle size.

Gas-dispersed systems are characterized by $\rho_t/\rho \gg 1$. Taking into account the Gasterstadt equation $\lambda_f = \lambda(1 + k\mu)$, Eq. (20) then has the form

$$\frac{Ka_f}{Ka} = \frac{\sqrt{(1 + k\mu)\left(1 + \frac{v}{v_T}\mu\right)}}{1 + \mu}. \tag{22}$$

Calculations based on this equation are difficult, becasue of the lack of reliable generalized data on the Gasterstadt coefficient k. Analysis of Eq. (22) showed that, with real values of k, v/v_t, and μ, the value of Ka_f/Ka varies monotonically and tends toward the constant limiting value $A = \sqrt{k(v/v_t)}$ as the concentration increases.

Calculations were made in order to illustrate this aspect of Eq. (22) (Fig. 1). The Gasterstadt coefficient for a channel 0.1 m in diameter and particles with $v_s = 0.8$ m/sec (solid line) and 5.5 m/sec (dash line) was determined from Pal'tsev's formula [5]. The ambient-medium (air) velocities are shown near the curves (Fig. 1). It was thus established that the presence of particles with $\rho_t \gg \rho$ in vertical dispersed flows can lead either to extinction of turbulence ($A < 1$) or to development of this process ($A > 1$), depending on the ratio of the quantities included in the complex A. When plotted on the coordinates Ka_f/Ka versus A, the results of calculations for different particles and carrier-medium velocities converge with rather good accuracy on individual constant-concentration lines (Fig. 2).

It is also of interest to compare the results obtained in calculating the relative turbulence intensity of dispersed and pure flows with experimental data on the relative heat-transfer rate between such flows and the wall (Fig. 3). Sergeev's dissertation [7], which we used for the comparison, gives direct experimental data on Nu_f/Nu and all the quantities included in complex A. The experiments were conducted in a vertical tube with a diameter of 78 mm and rapeseed was used as the dispersoid ($v_s \simeq 8$ m/sec). The calculated values of Ka_f/Ka are represented by the solid line and the experimental Nu_f/Nu by the dash line; curves 1 correspond to $v \simeq 17$ m/sec and curves 2 to $v \simeq 14$ m/sec. It follows from Fig. 3 that the attenuation of flow turbulence due to the properties of the discrete and continuous components leads to a decrease in heat-transfer rate at small flow concentrations. When the concentration is increased, the rate of external heat transfer begins to rise. This is due to the fact that the negative influence of the particles on flow turbulence becomes constant and the effective volume heat capacity of the flow and the perturbation of the boundary layer under the impact of particles from the wall (positive factors for heat transfer) increase with rising concentration.

Calculations made with Eq. (22) thus indicate qualitative agreement between data on the mechanics and heat transfer of vertical dispersed flows; the observed patterns enable us to explain the different effects of solid particles on the turbulence of a monodispersed flow.

NOTATION

- ρ is the density;
- v is the velocity;
- β is the true volume concentration;
- μ is the gravimetric flow concentration;
- b is the turbulence intensity;
- λ is the frictional drag coefficient;
- k is the Gasterstadt coefficient;
- A is the dimensionless complex; quantities without subscripts pertain to the continuous dispersed-flow component, those with the subscript T to the discrete component, and those with the subscript f to the total flow.

LITERATURE CITED

1. G. I. Barenblatt, Prikl. Mat. i Mekh., 17, No. 3 (1953).
2. G. I. Barenblatt, Prikl. Mat. i Mekh., 19, No. 1 (1955).
3. Z. R. Gorbis and F. E. Spokoinyi, Inzh.-Fiz. Zh., 15, No. 4 (1968).
4. A. S. Monin and A. M. Yaglom, Statistical Hydromechanics [in Russian], Part. 1, Nauka, Moscow (1965).
5. A. M. Dzyadzio and A. S. Kemmer, Pneumatic Transport in Grain-Processing Enterprises [in Russian], Kolos, Moscow (1967).
6. A. P. Yudin (editor), Pump Movement and Hydraulic Transport [in Russian], GÉI, Moscow–Leningrad (1963).
7. G. I. Sergeev, Candidate's Dissertation [in Russian], ITTF, Akad. Nauk UkrSSR, Kiev (1968).